Économie Publique

September 17, 2010

Notation

- L goods indexed by l; I consumers indexed by i; J Firms indexed by j
- $x^i = (x_1^i, ..., x_L^i)$ is consumption bundle of consumer i. $x^i \in \mathcal{C}^i = R_+^L$. Initial endowoment is $w^i \in R_+^L$
- $y^j = (y_1^j, ..., y_L^j)$ vector of production (inputs have negative sign). $y^j \in \mathcal{Y}^j \subset R^L$. Profits are given by py^j where the price vector $p \in R_+^L$
- θ^{ij} is the % of firm j owned by i.

Competitive Equilibrium

- A competitive equilibrium is given by a price system p^* and an allocation $(x^{*1}, ..., x^{*I}; y^{*1}, ..., y^{*J})$ such that
  - i) $p^* y^{*j} \geq p^* y^j$ for all feasible y^jin the production set.
  - ii) $U(x^{*i}) \geq U(x^i)$ for all x^i in the budget set.

$$B^i = \left\{ x^i : x^i \in \mathcal{C}^i \text{ and } p^* x^i \leq p^* w^i + \sum_{j=1}^{J} \theta^{ij} p^* y^{*j} \right\}$$

  - iii) Supply is equal to demand, for all good k

$$\sum_i x_k^{*i} = \sum_j y_k^{*j} + \sum_i w_k^i$$

Pareto Optimum

- An allocation $(x^1, ..., x^I; y^1, ..., y^J)$ is **feasible** iff $x^i \in \mathcal{C}^i$, $y^j \in \mathcal{Y}^j$ and

$$\sum_i x^{*i} \leq \sum_j y^{*i} + \sum_i w^i.$$

- An allocation is a **Pareto Optimum** if it is feasible and if it is not possible to find another feasible allocation that weakly improves welfare for all individuals (with a strict sign for one at least).

Welfare Theorems

- Theoreme 1: If $U^i(.)$ is strictly increasing, a competitive equilibrium is a Pareto optimum.
- Theoreme 2: If $U^i(.)$ is continuous, quasi concave, and strictly increasing, $\mathcal{Y}^j$ is convex, for any given Pareto allocation there is an assigment of wealth levels and a price vector such that firms and consumers maximize and markets clear.

Externality

- An EXTERNALITY is present whenever the well being of a consumer or the production set of a firm are directly affected by actions of other agents.
- Examples: Beekeeper-Orchard, Smokers, being literate, vaccine.
- We exclude effects through prices (pecuniary externality).

Example

- Two firms, two goods, one consumer who consumes x_1 and x_2.
- Production function of Firm 1 is:

$$y_1^1 = f^1(y_2^1)$$

- Production function of Firm 2 is:

$$y_2^2 = f^2(y_1^2, y_1^1, x_1)$$

The Pareto Optimum $(x_1, x_2, y_2^1, y_2^2, y_1^1, y_1^2)$. can be found as

$$Max\ \ U(x_1, x_2)$$

such that

$$-x_1 + y_1^1 + y_1^2 + w_1 \geq 0$$

$$-x_2 + y_2^1 + y_2^2 + w_2 \geq 0$$

$$-y_1^1 + f^1(y_2^1) \geq 0$$

$$-y_2^2 + f^2(y_1^2, y_1^1, x_1) \geq 0$$

If an interior solution, we have the following foc

$$\frac{\frac{\partial U}{\partial x_1} + \frac{\partial U}{\partial x_2}\frac{\partial f^2}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = -\frac{\partial f^2}{\partial y_1^2} = -\frac{1 + \frac{\partial f^2}{\partial y_1^2}\frac{df^1}{dy_2^1}}{\frac{df^1}{dy_2^1}}$$

That is, social marginal rate of substitution = to social marginal rates of trasformation

Note: $\frac{\partial f^2}{\partial y_1^2}$ and $\frac{df^1}{dy_2^1}$ are ≤ 0

Competitive Equilibrium

Prices are taken as given, but actions of the other are also taken as given.

$$Max \; p_1 y_1^1 + p_2 y_2^1 \quad such\ that \quad y_1^1 = f^1(y_2^1)$$

$$Max \; U(x_1, x_2) \quad such\ that \; p_1 x_1 + p_2 x_2 = R$$

where

$$R = p_1 w_1 + p_2 w_2 + \Pi$$

$$Max \; p_2 y_2^2 + p_2 y_1^2 \quad such\ that \quad y_2^2 = f^2(y_1^2, y_1^1, x_1)$$

Three FOC

$$-\frac{1}{\frac{df^1(y_2^1)}{dy_2^1}} = \frac{p_1}{p_2}$$

$$\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{p_1}{p_2}$$

$$-\frac{df^2(y_1^2, y_1^1, x_1)}{dy_1^2} = \frac{p_1}{p_2}$$

Question: can we say if production or consumption in CE is greater or smaller than optimal?

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Question: can we say if production or consumption in CE is greater or smaller than optimal?

Public Intervention

- Direct emission control.
- Allow negotiation between the parties (Coase Theorem works as long as property rights are well defined)
- Taxing activities that generate externalities. Need a lot of information
- Creating Competitive Markets for the right to engage in activities that generate externalities
- Firms Integration
- Pollution Abatement

Trading Rights

- Firm 1 must buy from Firm 2 the right to pollute. y_1^{12} is the quantity of pollution rights demanded by Firm 1. $\widehat{y}_1^{12}$ are the rights sold by Firm 2. In equilibrium, the two must coincide.

$$Max\ \ p_1 y_1^1 + p_2 y_2^1 - q_1^{12} y_1^{12}$$

$$y_1^1 = f^1(y_2^1) \ \text{ and } \ y_1^{12} = y_1^1$$

- Consumers must buy from Firm 2 the right to pollute. x_1^{12} is the quantity of rights to pollute demanded by the consumer. $\widehat{x}_1^{12}$ is the quantity sold.

$$Max\ \ U(x_1, x_2)$$

$$p_1 x_1 + p_2 x_2 + p_1^{12} x_1^{12} = R \quad and \quad x_1^{12} = x_1$$

- Firm 2 must sell the right to pollute

$$Max\ \ p_1 y_1^1 + p_2 y_2^1 + q_1^{12} \widehat{y}_1^{12} + p_1^{12} \widehat{x}_1^{12}$$

$$y_2^2 = f^2(y_1^2, \widehat{y}_1^1, \widehat{x}_1) \ \text{ and } \ y_1^{12} = y_1^1$$

Consumer is allocated the right to pollute

$$Max \ \ U(x_1, x_2)$$

$$p_1 x_1 + p_2 x_2 = R + p_1^{12}(\overline{x} - x_1^{12})$$

$$x_1 = x_1^{12}$$

Firm 1 is allowed to pollute $\overline{x}$ and must be paid in order NOT to pollute. Note that the BC becomes

$$p_1 x_1 + p_2 x_2 + p_1^{12} x_1^{12} = R + p_1^{12}\overline{x}$$

and foc are not affected (but income is affected)

Pigouvian Taxation

Competitive equilibrium with taxes t and τ is a price vector $(\overline{p}_1, 1)$ and allocation $(\overline{x}_1, \overline{x}_2, \overline{y}_1^1, \overline{y}_2^1, \overline{y}_1^2, \overline{y}_2^2)$ such that

$$Max \;\; (\overline{p}_1 - \tau)\, y_1^1 + y_2^1$$

$$s.t. \; y_1^1 = f^1(y_2^1)$$

$$Max \;\; U(x_1, x_2)$$

$$(\overline{p}_1 + t)\, x_1 + x_2 = \overline{R} + \overline{T}$$

Firm 2 same problem as before

Pigou Tax

$$\overline{p}_1 - \tau = -\frac{1}{\frac{df^1}{dy_2^1}}$$

$$\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \overline{p}_1 + t$$

$$-\frac{\partial f^2}{\partial y_1^2} = \overline{p}_1$$

Pigou Tax

Then,

$$\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} - t = -\frac{\partial f^2}{\partial y_1^2} = \tau - \frac{1}{\frac{df^1}{dy_2^1}}$$

So, it is enough to choose

$$t = -\frac{\partial f^2}{\partial x_1} \quad \text{and} \quad \tau = -\frac{\partial f^2}{\partial y_1^2}$$

That is, taxes are equal to marginal effects on production by 2 evaluated at the optimum.

Example (See J.J. Laffont, p. 197)

- I consumers, 1 firm
- Good 1 is not produced (4 units of endowment to each consumers)
- Good 2 is produced by the only firm using good 1.
- $y_2 = \sqrt{z_1} \quad z_1 \geq 0 \;\; y_2 \geq 0$
- Production of good creates an externality
- $U^i(x_1^i, x_2^i, y_2) = x_1^i + \log x_2^i - \frac{1}{2} \log y_2$

Example

- Solving for the competitive equilibrium we find that total production of good 2 is

$$y_2 = \sqrt{\frac{I}{2}}$$

individual consumption of good 2 is

$$x_2^i = \frac{1}{\sqrt{2I}}$$

- Solving for the Pareto Optima we obtain

$$y_2 = \frac{\sqrt{I}}{2} < \sqrt{\frac{I}{2}}$$

- Inefficiency aggravates when I is large

Finding Pareto Optima when there are more than 1 consumer

- One can show that Pareto optima are solutions to

$$Max \sum_{i=1}^{I} \alpha_i U^i(x_1^i, x_2^i, y_2)$$

subject to feasibility constraints, where α_i are the weights in the *Social Welfare Function*.

- By contradiction: Suppose that the allocation that solves the problem is not Pareto optimal. Then, we can find another allocation that is strictly better for an individual and not worse for the others. If $\alpha_i > 0$, this would imply that the initial allocation is not an actual maximum.
- One can also show that (under some conditions on U) every Pareto optimum is a solution to the above problem for some welfare weights $\alpha_i \geq 0$, $i = 1, .., I$.

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Detour on Externalities in Macroeconomics

- Knowledge externalities are often incorporated in Macro (Romer, Lucas, Farmer, Benhabib, etc).
- Assume a large number of identical firms.
  (*Private*) technology is

$$y_t = A_t k_t^a l_t^b$$

with

$$A_t = \bar{k}_t^{(\alpha - a)} \; \bar{l}_t^{(\beta - b)}$$

where $\bar{k}_t$ and $\bar{l}_t$ are the average uses of k and l in the economy.

$$a + b = 1, \;\; \alpha + \beta > 1$$

Detour on Externalities in Macroeconomics

- In a symmetric equilibrium, $k_t = \overline{k}_t$ and $l_t = \overline{l}_t$. Then, the (*social*) technology becomes

$$y_t = k_t^{\alpha} l_t^{\beta}$$

  and there are increasing returns.

- Evidence: Caballero Lyons (JME, 1994) regress industry output on industry inputs and aggregate output. Positive coefficient on aggregate output.

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Externalities in Macroeconomics

- Knowledge externalities are able to generate growth in the long run since MP of capital is not decreasing.
- They are also able to explain common long-run growth rates across countries despite persistently different rates of investment. See Klenow-Rodriguez-Clare (2005, Handbook of Ec. Growth)
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IR lead to Multiple Equilibria (Farmer and Benhabib, 1994)

- Recall that return to investment is a function of tomorrow marginal product of capital.
- If technology has increasing returns, higher stock of capital tomorrow increases return to investment.
- Suppose that agents expect higher stock of capital, therefore they expect return to investment to be high.
- As a result positive expectations lead to investment and are self-fulfilling (Animal Spirits).

Public Good

- Private Goods are excludable and rivalrous (i.e., use by one agent prevents other agents from using it)
- Pure Public Good (e.g., defense) is non-rivalrous and non-excludable
- A Club Good (e.g., a bridge) is excludable but non-rivalrous.
- Public Goods is another example where (in general) markets fail to allocate ressources efficiently.

Public Good

- x^i is private good.
- y is the public one.
- $w = \sum_{i=1}^{I} w^i$ is aggregate endowment of private good.
- The public good is produced using the private good according to

$$y = g(z)$$

where z is the amount of private good used in the production.

- $$\frac{dg}{dz} > 0, \quad \frac{d^2 g}{dz^2} < 0, \; z \geq 0$$

Public Good: Pareto Optima

We find $x^i \geq 0$, for all i, $z \geq 0$ and $y \geq 0$ that solve the following problem:
For given $\alpha_i \geq 0$,

$$Max \sum_{i=1}^{I} \alpha_i U^i(x^i, y)$$

$$w - \sum_{i=1}^{I} x^i - z \geq 0$$

$$g(z) - y \geq 0$$

Public Good: Pareto Optima

First Order Conditions with respect to x^i, for all i

$$\alpha_i \frac{\partial U^i}{\partial x^i} = \lambda$$

with respect to y,

$$\sum_{i=1}^{I} \alpha_i \frac{\partial U^i}{\partial y} = \mu$$

with respect to z,

$$\mu \frac{dg}{dz} = \lambda$$

Bowen-Lindahl-Samuelson Condition

One obtains,

$$\sum_{i=1}^{I} \frac{\frac{\partial U^i}{\partial y}}{\frac{\partial U^i}{\partial x^i}} = \frac{1}{\frac{dg}{dz}}$$

- Sum of MRS = MRT between the two goods (private and public)
- MRS is the amount of private good that you are willing to sacrifice to have one extra unit of public good.
- MRT is the amount of private good that must be given up if another unit of public good is provided.
- If LHS > RHS Planner should increase the public good and appropriately redistribute the cost.

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Kolm Triangle

- Suppose $I = 2$ and
$$y = z$$
- The resource constraint becomes
$$w = x^1 + x^2 + z$$
- w is given, the triplet (x^1, x^2, z) is a point in the triangle.
- Sum of distances to the sides is equal to w.

Kolm Triangle

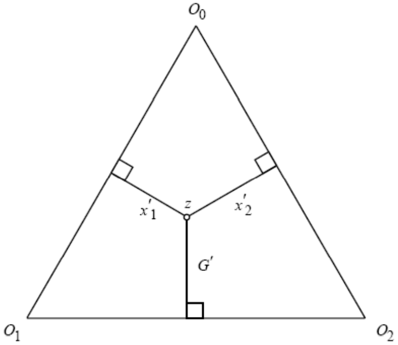


Fig. 1. A feasible allocation in a Kolm triangle.

Kolm Triangle

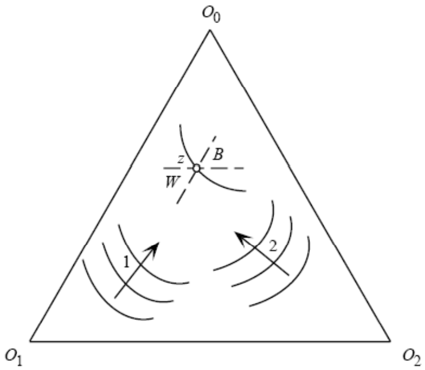


Fig. 2. The indifference maps in a Kolm triangle.

A Voluntary-Contribution Equilibrium

- Suppose the following game. Each consumer simultaneously proposes z_i
- The public good will be:

$$y = g(\sum_{i=1}^{I} z^i)$$

Voluntary Contribution

- Find the **best response function** of individual i.
- This is the contribution z^i that maximizes $i's$ utility given others' contribution

$$Max U^i(x^i, y)$$

$$y = g(z^i + \sum_{j \neq i}^{I} z^j)$$

$$x^i = w^i - z^i$$

Public Good

- The first order Condition is

$$\frac{\frac{\partial U^i}{\partial y}}{\frac{\partial U^i}{\partial x^i}} = \frac{1}{\frac{dg}{dz}}$$

- Individuals do not take into account the benefit to others of increasing $z \Rightarrow$ underprovision

Example

- Suppose
$$U^i(x^i, y) = \gamma \log y + \log x^i,$$
- $$y = z \text{ and } w^i = \frac{w}{I}$$

Egalitarian Pareto Optima

$$Max \sum_{i=0}^{I} \left(\gamma \log y + \log x^i\right)$$

$$y + \sum_{i=0}^{I} x^j = w$$

The BLS condition becomes

$$\sum_{i=1}^{I} \frac{\frac{\gamma}{y}}{\frac{1}{x^i}} = 1$$

which gives our egalitarian optimum:

$$y = \frac{w\gamma}{1+\gamma}$$

$$x^i = \frac{w}{(1+\gamma)\, I}$$

Voluntary contribution (Nash) equilibrium

Individual chooses z^i

$$Max \gamma \log y + \log x^i$$

$$x^i = \frac{w}{I} - z^i$$

$$y = z^i + \sum_{j \neq i}^{I} z^j$$

The first order condition is

$$\frac{\gamma}{z^i + \sum_{j \neq i}^{I} z^j} = \frac{1}{\frac{w}{I} - z^i}$$

Voluntary contribution (Nash) equilibrium

$$\frac{w\gamma}{I} = (1+\gamma)z^i + \sum_{j \neq i}^{I} z^j$$

- This gives the best response:

$$z^i = \frac{w\gamma}{I(1+\gamma)} - \frac{\sum_{j \neq i}^{I} z^j}{(1+\gamma)}$$

- This BR makes clear that i contributes less if others contribute a lot: Actions are **strategic substitutes**.

Voluntary contribution (Nash) equilibrium

Consider again the foc of individual i:

$$\frac{w\gamma}{I} = (1+\gamma)z^i + \sum_{j\neq i}^{I} z^j$$

We sum up over i

$$w\gamma = (I+\gamma)\sum_{i=1}^{I} z^i$$

Voluntary contribution (Nash) equilibrium

One then obtains:

$$y = \frac{w\gamma}{I + \gamma} \; < \; \frac{w\gamma}{1 + \gamma}$$

Given that contributions are identical,

$$x^i = \frac{w}{I} - \frac{y}{I} = \frac{w}{I + \gamma} \; > \; \frac{w}{(1 + \gamma)\, I}$$

- Individuals do not take into account the benefits to others of their contribution.
- Incentives to free-ride (increasing in I)
- Then, Nash Equilibrium leads to under-provision of public good and too much consumption of private good.

Voluntary contribution (Nash) equilibrium

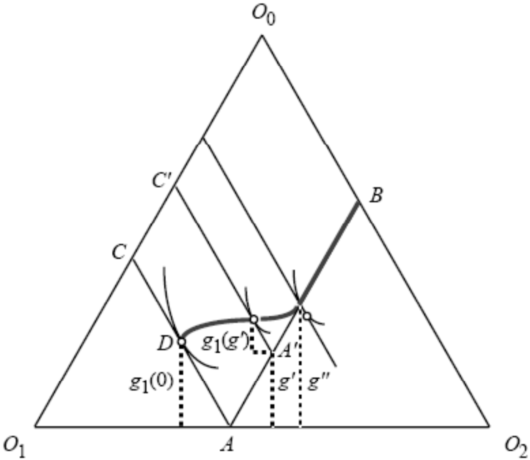
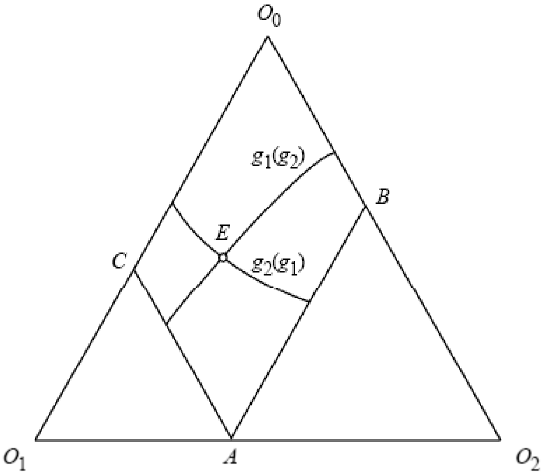


Fig. 6. Agent one's reaction function.

Voluntary contribution (Nash) equilibrium



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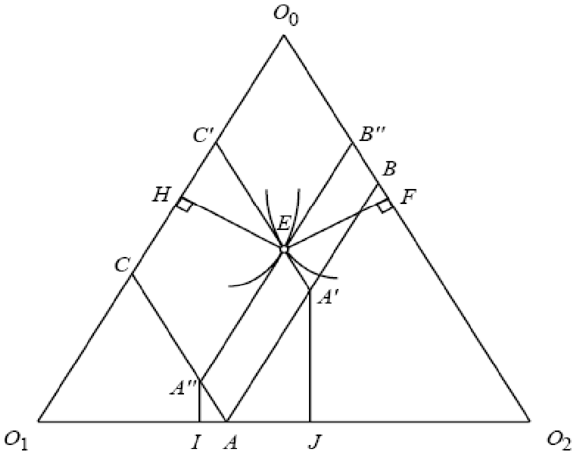
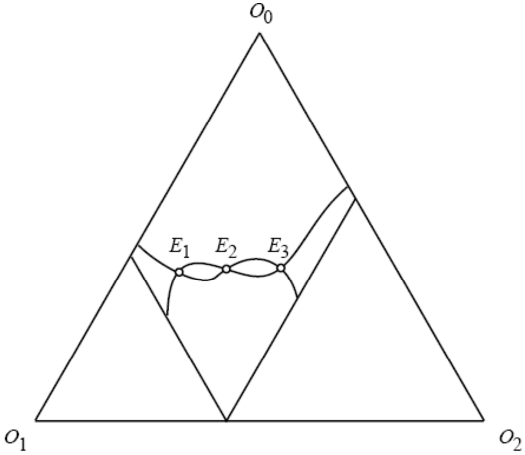


Fig. 5. A Nash equilibrium.

Nash outcome may not be unique



Again on Pareto-Optimal Allocations

- Look again at the BLS condition:

$$\sum_{i=1}^{I} \frac{\frac{\partial U^i}{\partial y}}{\frac{\partial U^i}{\partial x^i}} = \frac{1}{\frac{dg}{dz}}$$

- The MRS is a function of y and x^i, where x^i depends on the taxes paid for the public good.
- The optimal quantity of y changes as the burden of paying for y is shifted

Lindahl equilibrium

- We need to determine *at the same time* the quantity and how the public good is financed.
- We consider a *fictious* economy where each individual has a **personalized** price for the public good.
- The consumer solves

$$\underset{x^i \geq 0, y \geq 0}{Max} \; U^i(x^i, y)$$

$$w^i - x^i - p^i y \geq 0$$

- The demands for the two goods are $x^i(p^i)$ and $y^i(p^i)$. **NB: In equilibrium, we will need $y^i(p^i)$ to be the same for all i.**

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Lindahl equilibrium

Production of the public good is done by a competitive firm:

$$\underset{z\geq 0, y\geq 0}{Max} \left(\sum_{i=1}^{I} p^i\right) y - g^{-1}(y)$$

Marginal Cost = price

$$\left(g^{-1}\right)'(y) = \sum_{i=1}^{I} p^i$$

or

$$\frac{1}{g'(g^{-1}(y))} = \sum_{i=1}^{I} p^i$$

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$$\frac{1}{\sum_{i=1}^{I} p^i} = g'(z)$$

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Lindahl equilibrium

Input demand for the private good

$$z = \left(g'\right)^{-1}\left(\frac{1}{\sum_{i=1}^{I} p^i}\right)$$

Supply of public good is

$$\varsigma\left(\sum_{i=1}^{I} p^i\right) = g\left(\left(g'\right)^{-1}\left(\frac{1}{\sum_{i=1}^{I} p^i}\right)\right)$$

Input demands for the private good can also be written as

$$g^{-1}\left(\varsigma\left(\sum_{i=1}^{I} p^i\right)\right)$$

Lindahl equilibrium

A **Lindahl equilibrium** is a CE in this fictious economy. That is, a vector of prices $(p^{*1}, ..., p^{*I})$ and a corresponding allocation such that for all i demand of public good (given the personalized price) is the same

$$\varsigma(\sum_{i=1}^{I} p^{*i}) = y^i(p^{*i}).$$

Second, we need demand = supply of private good.

$$\sum_i^I x^i(p^{*i}) + g^{-1}\left(\varsigma(\sum_{i=1}^{I} p^{*i})\right) = \sum_i^I w^i$$

Characterization of Lindahl equilibrium

F.O.C. for each individual i implies that MRS = price ratio (private good is the numeraire)

$$\frac{\frac{\partial U^i}{\partial y}}{\frac{\partial U^i}{\partial x^i}} = p^{*i}$$

F.O.C. for the firm

$$\sum_{i=1}^{I} p^{*i} = \frac{1}{g'}$$

We then obtain the BLS condition

$$\sum_{i=1}^{I} \frac{\frac{\partial U^i}{\partial y}}{\frac{\partial U^i}{\partial x^i}} = \sum_{i=1}^{I} p^{*i} = \frac{1}{g'}$$

Characterization of Lindahl equilibrium

- If there were such a market, we would have a Pareto Optimum. But such MKT does not exist!
- GVT can compute these prices and force consumers to pay taxes

$$t^{*i} = p^{*i} y^*$$

- t^{*i} depend on preferences (increasing in the MRS)
- Suppose that population includes individuals that value y a lot and individuals that value y relatively less.
- Individuals with high valuation have an incentive to lie. Then, it is difficult to optimally spread the burden and choose y.

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Lindahl Equilibrium: Example

$$\underset{y \in [0, \frac{w}{I}]}{Max}\ \gamma \log y + \log x^i$$

$$x^i = \frac{w}{I} - p^i y$$

We obtain that optimal demand functions $x^i(p^i)$ and $y^i(p^i)$ are

$$y^i(p^i) = \frac{1}{p^i} \frac{\gamma w}{(\gamma + 1) I}$$

$$x^i(p^i) = \frac{w}{(\gamma + 1) I}$$

Lindahl Equilibrium: Example

$$\underset{y \in \left[0, \frac{w}{I}\right]}{Max} \gamma \log y + \log x^i$$

$$x^i = \frac{w}{I} - p^i y$$

We obtain that optimal demand functions $x^i(p^i)$ and $y^i(p^i)$ are

$$y^i(p^i) = \frac{1}{p^i} \frac{\gamma w}{(\gamma + 1) I}$$

$$x^i(p^i) = \frac{w}{(\gamma + 1) I}$$

Lindahl Equilibrium: Example

- Demand $y^i(p^i)$ must be the same for all i in a Lindahl equilibrium.
- Using

$$y^i(p^i) = \frac{1}{p^i}\frac{\gamma w}{(\gamma+1)I}$$

we have that p^{*i} is the same for all i.

- Use the foc of the firm $\sum_{i=1}^{I} p^{*i} = 1$ and obtain

$$p^{*i} = \frac{1}{I}$$

Lindahl Equilibrium: Example

- Knowing the individual function

$$y^i(p^i) = \frac{1}{p^i}\frac{\gamma w}{(\gamma+1)I}$$

  we obtain that public good provision in a Lindahl equilibrium is

$$y^i(p^{*i}) = \frac{\gamma w}{(\gamma+1)}$$

- Individuals pay ${p^*}^i y^*$ for the public good.
- If individuals have different γ_i prices will be different.

Lindahl Equilibrium when initial endowment is A

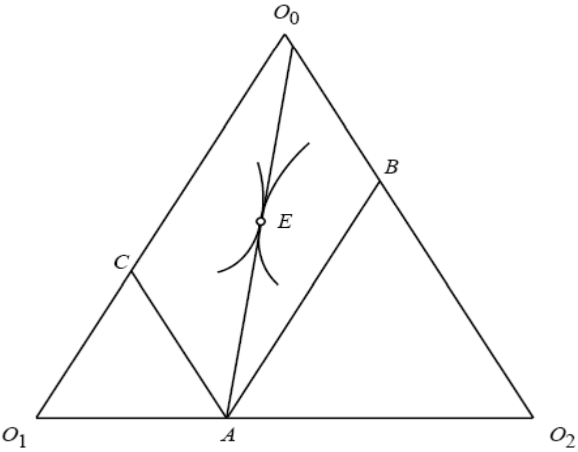

Fig. 14. Lindahl Equilibrium.

Majoritarian Voting

- Economy with I consumers. Suppose $y = z$ and $w^i = 1$
- To make politics interesting, differences in preferences (utility is quasi-linear)

$$U(x^i, y) = \theta^i \log y + x^i$$

where $\theta^i \in [0, 1]$.

Majoritarian Voting vs Pareto

At the Pareto-optimum we need

$$\sum_{i=1}^{I} \frac{\frac{\partial U^i}{\partial y}}{\frac{\partial U^i}{\partial x^i}} = \frac{1}{\frac{dg}{dz}}$$

That is,

$$\sum_{i=1}^{I} \theta^i = y$$

Majoritarian Voting vs Pareto

Knowing that

$$\frac{\frac{\partial U^i}{\partial y}}{\frac{\partial U^i}{\partial x^i}} = p^{*i}$$

Lindahl prices

$$p^i = \frac{\theta^i}{\sum_{1=j}^{I} \theta^j}$$

The implicit tax is

$$t^i = p^i y = \theta^i$$

Majoritarian Voting

- Suppose there is a vote to decide the *uniform* (lump sum) tax to finance the public good.
- Individual BC is

$$x^i = 1 - t$$

- Government BC is

$$tI = y$$

- Individual i has *induced preferences* over the uniform tax t:

$$\theta^i \log tI + (1 - t).$$

Majoritarian Voting

Plot indirect utility as a function of t

$$\theta^i \log tI + (1 - t).$$

- Note that it is strictly concave. Hence, it is single peaked with peak at $t = \theta^i$.
- (If $y = g(z)$ is convex, indirect utilities may not be single peaked.)
- How does voting work? Suppose that majority rule is used.

DETOUR on Condorcet Cycles

- Majority rule may create problems when the number of alternatives is at least 3. It fails to generate a well-defined choice.
- Consider three alternatives. $t \in \{0, 0.5, 1\}$
- Alex's ranking is (from the best to the worst option): 0, 0.5 and 1.
- Bob's ranking is 1, 0 and 0.5
- Carl's ranking is 0.5, 1 and 0.

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- Suppose that individuals vote **sincerely** (that is, in each vote they say "yes" to the alternative that gives them the highest utility).
- Suppose that agenda is **open**: individuals vote over pairs of policy alternatives. The winner in one round is put against a new alternative. **All** feasible alternatives are considered.
- What's the alternative that is preferred under majority rule? Policy 0 is majority-preferred to 0.5, which is preferred to 1. But 1 is preferred to 0. There is a cycle!
- A **Condorcet Winner** (a policy that beats any other policy in a pairwise vote) does not exist.

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Condorcet Cycles

- This problem is widespread and proper to other voting rules if the number of alternative is large enough.
- This problem does not arise if preferences are well-behaved.
- For example, if preferences are single peaked and I is odd majority rule selects a unique alternative.
- **The alternative preferred by the median is selected under majority rule.**
- Who is the median? Given I individuals, call 1 the individual with the lowest ideal point, 2 the one with the second lowest ideal point and so on, so that I is the individual with the highest ideal point. Assume I odd, the median is individual

$$\frac{I+1}{2}$$

Majoritarian Voting

Since preferences are single peaked in our example, the median voter applies

$$t = \theta^{MED}$$

$$y = I\theta^{MED}$$

Majoritarian Voting

Optimum was

$$\sum_{i=1}^{I} \theta^i = y$$

Majority voting leads to

$$y = I\theta^{MED}$$

How does the majoritarian solution compare with the Samuelson solution?

$$\sum_{i=1}^{I} \theta^i \overset{?}{\lesseqgtr} I\theta^{MED}$$

We have that there is overprovision of y iff the average θ is lower than the median

$$\frac{\sum_{i=1}^{I} \theta^i}{I} < \theta^{MED}$$

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