1- exam the 8

Revision for Final – 2014

1. Competitive equilibrium – exchange

Suppose there are only two goods (bananas and fish) and 2 consumers (Annie and Ben) in an exchange economy. Annie has a utility function $u_A(b, f) = b^2 f$ where b is the amount of bananas she eats and f is the amount of fish she eats. Annie has an endowment of $w_A^b = 7$ bananas and $w_A^f = 3$ kilos of fish. Ben has a utility function $u_B(b, f) = 2f + 3\log(b)$ and endowments of $w_B^b = 0$ bananas and $w_B^f = 4$ kilos of fish. Assume the price of bananas is 1.

(a) Write down the definition of a competitive equilibrium for this economy.

(b) Solve for the competitive equilibrium fish price p_f^* and the competitive equilibrium allocation of fish and bananas between Annie and Ben.

2. Social planner (Pareto)

Consider the following social welfare planner problem (in the labor market context).

$$\max_{\{c,n,l,y\}} \log c + \log l$$

$$\text{s.t.} \begin{cases} l + n = 24 \\ c + 2 = y \\ y = \sqrt{n} \\ c, l, n, y \geq 0 \end{cases}$$

Characterize the optimal allocation for the social welfare planer problem. Discuss the link with the competitive equilibrium (The firm in the economy is producing at $f = n^{1/2}$).

3. Pareto Efficient allocations

Consider a two-good, two-person exchange situation. There is one (perfectly divisible) unit of each of two goods, food (F) and clothing (C). There are two consumers, Ann and Betty. Denoting their consumption quantities by F_A for Ann's consumption of food etc., their utility functions are

$$U_A(F_A, C_A) = F_A\, C_A\,, \qquad U_B(F_B, C_B) = (F_B)^2\, C_B\,.$$

Find expressions for their marginal rates of substitution. For Pareto efficiency, the two MRSs should be equal. This gives you one equation linking the four quantities F_A, C_A, F_B, C_B. There are two other equations, namely the "material balance" requirements that for each good, the amount allocated to the two consumers should add up to the total available:

$$F_A + F_B = 1, \qquad C_A + C_B = 1\,.$$

Use these to solve for C_A, F_B, and C_B, each of them as a function of F_A.

Show C_A as a function of F_A in the Edgeworth box diagram (as in Lecture) with F_A on the horizontal axis and C_A on the vertical axis. Remember that the total amounts of each are 1, so only the portion of the graph in the unit square ($0 \le F_A \le 1$, $0 \le C_A \le 1$) has economic significance. With Ann's quantities measured from the usual origin, Betty's quantities are automatically the residuals, as if they were measured from an origin at the diagonally opposite point $(1, 1)$ and axes going to the left and downward. In this box, note that the graph you drew lies above the 45-degree line? What is the intuition for this?

4. Competitive equilibrium

Suppose that in the above example, Ann initially owns the unit of food and Betty owns the unit of clothing. They can exchange these goods, acting as price-takers. Denote the price of food by P_F and the price of clothing by P_C.

4.1 write down the demand functions
4.2 find the price ratio for the CE

5. Competitive equilibrium

Assume that the agents have Cobb-Douglas preferences,

$$U^A(x_1, x_2) = a \ln x_1 + (1 - a) \ln x_2$$
$$U^B(x_1, x_2) = b \ln x_1 + (1 - b) \ln x_2,$$

and that the endowments are $e^A = (2, 0)$ and $e^B = (0, 1)$. In words, agent A is a seller of good 1 and a buyer of good 2 and agent B is the other way around.

Characterize the equilibrium

6. Cost Minimization

The technology of the firm can be represented by the function:

$$Y = F(X_1, X_2) = A + B\ln(X_1) + C\ln(X_2)^2$$

A, B, C are parameters and price of X_1, X_2 are W_1, W_2 respectively. Assume an interior solution.

a. Set up the minimization problem.
b. Find the TRS and interpret.
c. Find the firm conditional factor demand functions.
d. Calculate the cost function $C(W_1, W_2, Y)$.
e. Calculate the supply function $F(W_1, W_2, Y)$.
f. Calculate the profit function $\pi(W_1, W_2, Y)$.

7. Profit Maximization

A firm has the following production function:

$$F(X_1, X_2) = A + BX_1^{-1} + CX_2^{-1}$$

A, B, C are parameters and price of X_1, X_2 are W_1, W_2 respectively. Assume an interior solution.

a. Write the maximization problem for the firm.
b. Solve for the factor demand.
c. Which conditions on parameters A, B & C we must impose to have a solution?
d. Find the supply function for the firm $F(P, W_1, W_2)$.
e. Find the profit function for the firm $\pi(P, W_1, W_2)$.

8. Kuhn Tucker conditions

Lets condiser the following maximization problem.

$$F(x_1, x_2) = x_1 x_2$$
$$\text{s.t}$$
$$x_1 + x_2 \leq 100 \ \&$$
$$x_1 \leq 40,$$
$$x_1 \ \& \ x_2 \geq 0$$

8.1 Write the maximization problem
8.2 Write the KT conditions
8.3 Solve

9. Consumer theory

A consumer has utility function

$$u(x_1, x_2) = \left(x_1^{1/2} + 2x_2^{1/2}\right)^2.$$

9.1 Find the marshallian demand function
9.2 Find the consumer indirect utility function