Additional questions for Chapter 3:

1. **Profit function:**

   Consider the following profit function.

   $$\pi(p, \mathbf{w}) = p^{-(\frac{1}{\beta-1})}(w_1^{\gamma} + w_2^{\gamma})^{(\frac{\beta}{\gamma(\beta-1)})}\beta^{-(\frac{\beta}{\beta-1})}(1-\beta)$$

   where $\gamma = \frac{\rho}{(\rho-1)}$ and $0 < \beta < 1$.

   Derive the supply functions corresponding to this profit function.

2. **Profit maximization with 2 outputs**

This problem concerns a price-taking firm that produces two outputs, goods 2 and 3, using a single input, good 1. Its implicit production function (aka transformation function) is:

$$F : \Re^3 \to \Re : F(y_1, y_2, y_3) = 2(y_2)^2 + 3(y_3)^2 + y_1,$$

so that any vector (y_1, y_2, y_3) *satisfying* $y_1 \leq 0$ *and* $2(y_2)^2 + 3(y_3)^2 + y_1 \leq 0$ *is technologically feasible. Moreover, its production set satisfies the free disposal assumption.*

1 *Does the technology display nonincreasing returns to scale? Show your work and explain.*

2 *Write the Maximization Problem.*

3 *Obtain the supply function and profit function. (assuming price-taking in all input and output markets).*

4 *Verfy that Hotelling's Lemma applies with all three commodities, input and outputs.*

5 *Show that the simple law of supply applies to both outputs and that the simple law of demand applies the single factor of production.*

3. Recapitulative question:

General CES Production.

For this problem set you are going to work with a more general version of CES production than presented in class. Let the production function $f : \Re^2_+ \to \Re$ be defined by:

$$f(x) = \left(a_1 (x_1)^\rho + a_2 (x_2)^\rho\right)^{\alpha/\rho},$$

where $a_1 > 0$, $a_2 > 0$, $\alpha > 0$ and $\rho \in (-\infty, 0) \bigcup (0, 1)$.

1. Carefully state this production technology as a transformation function. Include all appropriate restrictions on independent variable values.

2. Specify the general input requirement set as a function of output, y.

3. Carefully show that this technology is monotone.

4. Provide an exact characterization of all parameter value combinations such that the consequent production technology has decreasing returns to scale. Show your work (i.e., justify your characterization).

   One way of providing an exact characterization is with set notation. For example with the subset of real numbers $\widehat{R} = \rho \in (-\infty, 0) \bigcup (0, 1)$, you could define this set with the form,

   $$\left\{(a_1, a_2, \alpha, \rho) \in \Re^3_{++} \times \widehat{R} \mid \ldots \ldots \right\}.$$

5. Is this production technology homothetic (perhaps for only some parameter value combinations)? Fully justify your answer.

6. Provide the Technical Rate of Substitution of input 1 in terms of input 2 for this technology as a function in the form $TRS_{1,2} = \ldots$. Show your work, starting with the production function provided above.

7. Provide the gradient for this production function. Show your work.