Additional questions for Chapter 10

Equivalent and Compensating Variation with Cobb-Douglas Preference

You are going to work with Cobb-Douglas preference as represented by the utility function $u : \Re^L_+ \to \Re : u(x_1, \dots, x_L) = \prod_{j=1}^{L} x_j^{\alpha_j}$ where $\alpha_j > 0$, $j = 1, \dots, L$, and $\sum_{j=1}^{L} \alpha_j = 1$. I solved the $UMAX$ problem for this utility function in class, but without the restriction that $\sum_{j=1}^{L} \alpha_j = 1$. With that restriction we have the Walrasian demand function,

$$\widehat{x}(p, w) = \begin{pmatrix} \dfrac{\alpha_1 w}{p_1} \\ \vdots \\ \dfrac{\alpha_L w}{p_L} \end{pmatrix},$$

and the indirect utility function function,

$$v(p, w) = \prod_{j=1}^{L} \left(\frac{\alpha_j w}{p_j} \right)^{\alpha_j}.$$

(These two functions can also be obtained from the problem set 2 answer key for Stone-Geary by setting $b = 0$ and $\bar{\alpha} = 1$.)

<u>Developing some functions</u>. The first two parts of this problem set require you to develop some general functions.

1. Find the expenditure and Hicksian demand functions for this representation of Cobb-Douglas. There are at least three methods for finding these functions. Which method you use is up to you. (With one method you can completely avoid calculus, if that is important to you.)

2. For a general change in prices from p^a to p^b, with price vectors $p^a, p^b \in \Re^L_{++}$, $p^a = (p^a_1, \dots p^a_L)$ and $p^b = (p^b_1, \dots p^b_L)$, develop the Equivalent Variation and Compensating Variation functions for this change. Please state these as functions and simplify as much as possible. (Hint: Consider that Cobb-Douglas type preference may be part of a much larger preference family, and that I presented general forms of both EV and CV for two such families. This may help you validate your results.)

Applying EV and CV functions. The last three parts of this problem set are concerned with three different scenarios involving p^a and p^b.

3. Suppose that $p_1^b < p_1^a$ and $p_j^b = p_j^a$ for all $j = 2, \ldots, L$ (the price of good one falls and all other prices remain the same).

   a. If possible, simplify your expressions for EV and CV (from part 2) based on this additional information.

   b. Based on algebraic reasoning determine the signs of these two (new) expressions. Then explain these signs based on the economics of the price change.

   c. Based on algebraic reasoning determine the relative magnitudes of these two expressions (determine which, if any, is bigger in absolute magnitude). Then use this information and that obtained in part 3.b to determine whether good one is normal, inferior or has no wealth effect (based on information developed in class).

   d. A story: A particular policy study requires finding the numerical value of EV. Unfortunately, the job is independently assigned to absent-minded Archer and to lazy Lew. Archer comes up with the value for CV, and Lew comes up with the value for ΔCS. Thus, Archer's error rate is $(CV - EV)/EV$, to be written e_a, whereas Lew's error rate is $(\Delta CS - EV)/EV$, to be written e_ℓ. What are the signs of these error rates? How do the absolute values of Archer's and Lew's error rates compare?

4. Suppose that $p_j^b > p_j^a$ for all $j = 1, \ldots, L$ (the prices of all goods rise).

   a. If possible, simplify your expressions for EV and CV (from part 2) based on this additional information.

   b. Based on algebraic reasoning determine the signs of these two (new) expressions. Then explain these signs based on the economics of the price change.

   c. Based on algebraic reasoning determine the relative magnitudes of these two expressions (determine which, if any, is bigger in absolute magnitude). Then use this information and that obtained in part 4.b to determine whether all the goods taken together in some loose aggregate sense are normal, inferior or have no wealth effect (based on the same general thinking that you used for part 3.c).

5. Suppose that $p_1^a = 1$, $p_2^a = 2$, $p_1^b = 2$, $p_2^b = 1$ and $p_j^b = p_j^a$ for all $j = 3, \ldots, L$ (The price of good one rises from 1 to 2, price good two falls from 2 to 1, and all other prices remain the same).

   a. If possible, simplify your expressions for EV and CV (from part 2) based on this additional information.

   b. Based on algebraic reasoning determine the signs of these two (new) expressions in terms of the α_1 and α_2 values. Also provide an economic intuition for your result in terms of the economic role of the α values in the utility function.