

**Additional question for Chapter 17**

1)

A pure exchange economy has 1000 type A participants and 1000 type B participants. There are two goods, named goods 1 and 2. Each type A has an initial endowment of 9 units of Good 1 and no Good 2. Each type B has an initial endowment of 16 units of Good 2 and no good 1. Each Type A consumer has the utility function

$$U_A(x_1, x_2) = x_1 + 2\sqrt{x_2}.$$

Each Type B consumer has utility function

$$U_B(x_1, x_2) = x_2 + 2\sqrt{x_1}.$$

Let good 1 be the numeraire and let  $p$  be the price of good 2.

- i) Find the excess demand function of each type of consumer for good 1. For what prices are there corner solutions where one or both consumers would choose to consume only one good?
  
  
  
  
  
  
  
  
  
  
- ii) For what price or prices is there a competitive equilibrium in which both consumers consume positive amounts of each good?
  
  
  
  
  
  
  
  
  
  
- iii) Find all of the competitive equilibria for this economy. (Hint: Pay attention to corner solutions. A really outstanding answer would not only find all of the equilibria but show that there are no others.)

Question 2)

A pure exchange economy has 5,000 consumers and 3 goods  $X$ ,  $Y$ , and  $Z$ . Every consumer  $i$  in the economy has utility function

$$U_i(x_i, y_i, z_i) = (x_i^{1/2} + y_i^{1/2} + z_i^{1/2})^2,$$

where  $x_i$ ,  $y_i$ , and  $z_i$  are  $i$ 's consumption of  $X$ ,  $Y$ , and  $Z$ , respectively. There are 3 types of consumers. There are 2,000 Type 1 consumers and each has an initial endowment of 4 units of good  $x$  and 0 units of goods  $y$  and  $z$ . There are 1000 Type 2 consumers and each has an initial endowment of 2 units of good  $y$  and 0 units of goods  $x$  and  $z$ . There are 2000 Type 3 consumers and each has an initial endowment of 1 unit of good  $z$  and 0 units of goods  $x$  and  $y$ .

A) Let Good  $X$  be the *numeraire* good and find competitive equilibrium prices for Goods  $Y$  and  $Z$ .

B) Compare the "income" (value of initial endowment) of a Type 1 consumer with that of a Type 2 consumer and with that of a Type 3 consumer. What is the total value of initial endowments at competitive equilibrium prices?

C) In competitive equilibrium, how much Good  $X$  does a Type 1 consume? How much Good  $X$  does a Type 2 consume? How much Good  $X$  does a Type 3 consume?

D) A natural disaster strikes the suppliers of Good 1 and reduces the initial endowment of each Type 1 consumer from 4 units of good  $X$  to 1 unit of good  $X$ . What are the prices in the new competitive equilibrium?

Question 3)

Consider an economy in which endowments are as in the previous problem, before the natural disaster, but utility functions are

$$U_i(x_i, y_i, z_i) = (x_i^{-1/2} + y_i^{-1/2} + z_i^{-1/2})^{-2}.$$

- A) Find the competitive prices for this economy.
- B) Compare the “income” (value of initial endowment) of a Type 1 consumer with that of a Type 2 consumer and with that of a Type 3 consumer.
- C) Suppose that in this economy a natural disaster strikes the suppliers of Good 1 and reduces the initial endowment of each Type 1 consumer from 4 units of good  $X$  to 1 unit of good  $X$ . What are the prices in the new competitive equilibrium? In the new equilibrium, compare the value of the endowment of a Type 1 consumer with that of a Type 2 consumer and that of a Type 3 consumer.
- D) (For extra credit) In this case would Type 1’s benefit from the disaster? Can you say anything more general about when a type would gain (lose) if the endowments of everyone of that type were to increase (decrease)?

Question 4)

Consider a two-person economy and solve for a Walrasian equilibrium. Let consumers 1 and 2 have identical CES utility functions,

$$U^i(x_1, x_2) = x_1^p + x_2^p, \quad i = 1, 2$$

Where  $0 < p < 1$ . Initial endowments are  $e^1 = (1, 0)$  and  $e^2 = (0, 1)$ . Find the Walrasian equilibrium.